Expressions, Equations, Inequalities, and Functions							
Construct	Write, read, and evaluate expressions in which letters stand for numbers. • Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as 5 – y.* • Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, difference); view one or more parts of an expression as a single entity. *For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.*	Use variables to represent numbers and write expressions when solving a mathematical or real-world problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set. Identify expressions that represent a quantity in terms of its mathematical context. Identify parts of an expression, such as terms, factors, coefficients, exponents, and base.	Construct a simple linear equation to model a relationship between two quantities. Determine the rate of change and initial value of the equation from a description of a relationship or from two (*x*, *y*) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear equation in terms of the situation it models, and in terms of its graph or a table of values. (*For example, given that a taxi company charges a base rate of $1.00 plus 10 cents per minute, and y is the cost, in dollars, of using the taxi for x minutes, determine the slope-intercept form of the equation that describes this situation.)*	Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.*	Construct simple linear inequalities of the form $px + q > r$ or $px + q < r$, where *p*, *q*, and *r* are specific rational numbers. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make.*	Construct exponential functions, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). Identify expressions that represent a quantity in terms of its mathematical context. • Identify parts of an expression, such as degree, index, complex numbers, imaginary part, real part, and log. *For example, interpret $(x + 1)(2x^2 + 3x)^3$ as the product of (x + 1) and a factor not depending on (x + 1).*	Write a function, using function notation, that describes a relationship between two quantities. • Construct a function to model a linear relationship. • Combine standard function types using arithmetic operations. • Compose functions. Describe the domain of the composed function with respect to the domains of original functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.*

			Determine the equation of a line given two points on the line and the slope. Construct a linear equation in one variable to model a verbal description of a real-world situation. (*For example, write an equation that could be used to determine the amount of sales tax collected (x), in dollars, given that a company earned a total of $1,696 in one day and the sales tax rate is 6%. the cash register, counting both the sale of goods and the sales tax of 6%.)*				
Simplify and Rewrite	Simplify/evaluate numerical expressions using order of operations. Understand that expressions that simplify to zero in the denominator are	Simplify/evaluate algebraic expressions using basic operations and order of operations. *For example, evaluate* $x^3 - y(x-z)^2$ *when* $x = -2$, $y = -3$, *and*	Apply properties of operations as strategies to add, subtract, factor, and expand linear and polynomial expressions with rational coefficients, including simplifying the result.	Choose and produce an equivalent form of an expression or equation to reveal and explain properties of the quantity represented by the expression or equation.	Simplify basic expressions with integer exponents (i.e. basic laws of exponents), such as $3x^8/15x^{12}$. Simplify simple rational and radical expressions (limited to square roots), such as	Use the structure of an expression (including expressions as part of equations and inequalities) to identify ways to simplify and rewrite it. • Factor expressions (e.g.,	Rewrite logarithmic expressions as exponential expressions and exponential expressions as logarithmic expressions. Understand and use the properties of logarithms to

	undefined.	$z = -1$.	*For example, apply the distributive property to the expression $(2 + x)(3x^2 + 2x - 5)$ to produce the equivalent expression $3x^3 + 8x^2 - x - 10$; factor the expression $24x + 18xy$ to produce the equivalent expression $6x(4 + 3y)$; apply properties of operations to $2x^2 + y + 2y - 5x^2$ to produce the equivalent expression $-3x^2 + 3y$, or recognize that $-4x^3y$ and $5yx^3$ are like terms.*	• Factor a simple quadratic equation to reveal the zeros. • Simplify simple rational and radical equations. • Factor a simple binomial or trinomial expression. • Combine basic rational expressions with unlike denominators.	$(1 - x)/(x^2 - x)$, $\sqrt{8x^7}$, *or* $3\sqrt{18x^3} - 5x\sqrt{50x}$. Find and simplify basic intersections and unions of sets and intervals and express the results using appropriate notation, including roster notation, set-builder notation, and interval notation.	trinomials with leading coefficient of 1 or greater, quadratic, quadratic in form, difference of squares, and sum/difference of cubes) when given roots, based on pattern recognition, or by grouping. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$ and as $(x + y)(x - y)(x^2 + y^2)$.* • Simplify rational and radical expressions, including those with rational exponents, including rational expressions that result in division of or by zero. *For example, simplify* $\left(8x^6y^{-9}\right)^{\frac{2}{3}}$ *or* $\left(\frac{4x^{-2}y^3}{8x^4y^{-1}}\right)^2$. • Rearrange literal expressions to highlight a quantity of interest.	simplify logarithmic numeric expressions and to identify their approximate value. Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where *a*, *c*, and *d* are numbers and the base *b* is 2, 10, *e*, or other simple expressions (such as $(1 + r)$ when *r* is given); evaluate the logarithm using technology.

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Solve (Linear)	Solve linear equations and linear inequalities in one variable. • Identify linear equations in one variable with one solution, infinitely many solutions, or no solutions. Determine which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers). • Solve linear equations and linear inequalities with rational number coefficients, including equations and inequalities whose solutions require expanding expressions using the distributive property and collecting like terms.	Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations to solve problems by reasoning about the quantities. • Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?* Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are rational numbers.	Solve basic literal equations in one variable (one copy of the variable being isolated). Include evaluation of formulas for a specific variable (*for example, convert between temperatures expressed in Fahrenheit and Celsius*).	Solve compound inequalities in one variable. Solve literal equations that require factoring (two copies of the variable being isolated).	Solve real-world problems involving equations and inequalities in one variable. *Include equations arising from linear functions (e.g., percent increase/decrease, simple interest, mixture (for example, given that in a chemistry class, 9 liters of a 4% silver iodide solution will be mixed with a 10% silver iodide solution, to result in a 6% solution, determine the number of liters of the 10% solution that are needed.), uniform motion).*	Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept, and the real-world meanings of these values. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.*

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Solve (Nonlinear)	Solve equations in one variable that are quadratic and quadratic in form. • Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. • Recognize when the quadratic formula gives non-real solutions.	Solve real-world problems involving given expressions and equations that are: • Simple quadratic (*For example, given that the number of milligrams (M) of a medication in the blood stream of a 132-lb patient t hours after a 3-milligram dose has been administered can be approximated by* $M = \frac{1}{2}t^2 + \frac{5}{2}t$ *, determine the number of hours after administration that there will be about 2 mg in the bloodstream.)* • Rational (e.g., proportions (one quantity) (*for example, given that a label printer prints 3 pages of labels every 5.0 seconds, determine the number of seconds it take the printer to print 111 pages of labels.)*, variation (*for example, given that the number of amperes (A) of electric current in a circuit varies directly with the number of volts (V) of voltage applied, and that the current is 3 A when 19 V are applied to a circuit , determine the current in the circuit when 12 V are applied.)*)	Solve absolute value inequalities and other nonlinear inequalities to determine the solution set.	Solve equations in one variable (e.g., exponential, logarithmic, polynomial, absolute value, rational, and radical) by factoring and by other methods, and recognize how extraneous solutions may arise. Use the quadratic formula to solve an equation that results in non-real solutions, and write solutions as $a \pm bi$ for real numbers a and b.	Solve real-world problems involving expressions and equations that are: • Rational (e.g., work rate problems *(for example, given that conveyor belt X can move 1,000 boxes in 6 minutes, and conveyor belt Y can move 1,000 boxes in 9 minutes; when conveyor belt Z is added and all three are used simultaneously, 1,000 boxes are moved in 3 minutes, determine the number of minutes it would take conveyor belt Z to move 1,000 boxes on its own.)*, proportions (two quantities), variation) • Radical (e.g., pendulum swing, highway curves, view on a horizon) *(for example, given that the number of centimeters (d) that a spring is compressed from its natural, uncompressed position is given by the formula* $d = \sqrt{\frac{2W}{k}}$*, where W is the number of joules of work done to move the spring and k is the spring constant, determine the amount of work, in joules, needed to move a spring 4 centimeters when it has a spring constant of 0.4.)* • Quadratic (e.g., motion, mixture, work) *(For example, an express train travels 190 miles between two cities in 5 hours; during the first 90 miles of the trip, the train traveled through mountainous terrain and traveled the remainder of the trip on level terrain; given that the train traveled 20 miles per hour* ***slower*** *through the mountainous terrain than on level terrain, determine the speed of the train on level terrain.* (continued)	

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				• Exponential and logarithmic (including evaluating complicated expressions such as $\frac{\ln(3)}{\ln(2)-\ln(5)}$) *(For example, given the formula* $A = P\left(1+\frac{r}{n}\right)^{nt}$, *determine, to the nearest tenth of a year, the amount of time it takes a $3,200 investment to double when it is invested at an 8% interest rate and compounded semiannually.)* •	
Systems	Analyze and solve pairs of simultaneous linear equations. • Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. • Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations.	Analyze and solve real-world problems involving 2×2 linear systems. (*For example, given that there were 40,000 seats occupied at a baseball game, tickets for reserved seats cost $14 and tickets for general-admission seats cost $4, and ticket sales generated a total of $300,000, determine the number of seats of each type that were occupied.)* • Write two linear equations in two variables, given a verbal description of a context. • Interpret coefficients within given linear equations, in terms of the context.	Analyze and solve real-world problems involving 2×2 linear systems. (*For example, given that the length of a rectangular garden is 6 feet more than 3 times the width, and a total of 68 feet of fencing is required to border the garden, determine the length and width of the garden.)*	Solve simple 3×3 systems of linear equations. Analyze and solve simple real-world problems involving 3×3 linear systems.	Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line* $y = -3x$ *and the circle* $x^2 + y^2 = 3$.
Formulas and Geometry	Solve real-world problems by evaluating given formulas for specific variables, with attention to proper units. • Determine area of two-dimensional figures (e.g., triangles, quadrilaterals, other polygons, and	Solve real-world problems by evaluating given formulas for specific variables, with attention to proper units. • Determine volume of three-dimensional figures (e.g., right rectangular prisms, cones, cylinders,	Apply the Pythagorean Theorem in real-world and mathematical problems. • Determine unknown side lengths in right triangles in (limited to two dimensions). • Determine the distance between two points in a		Apply models and formulas to solve simple design problems (e.g., maximize area when given a fixed perimeter, such as when building a fence).

	circles), including area of compound figures by decomposing into triangles, rectangles, and other shapes. • Determine perimeter of two-dimensional figures (e.g., triangles, quadrilaterals, other polygons, and circles), including perimeter of compound figures by using properties of shapes to determine missing side lengths.	spheres) with whole number or fractional dimensions, including volume of compound figures composed of two non-overlapping three-dimensional figures. • Determine surface area of three-dimensional figures (e.g., right rectangular prisms, triangular prisms) with whole number or fractional dimensions, including surface area of compound figures composed of two non-overlapping three-dimensional figures. • Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to determine the surface area of these figures.	coordinate system.	
Functions, Relations, and Relationships				
Identification	Distinguish between relations and functions, identifying when a relation is a function. Understand that a function is a rule that assigns to each input exactly one output, and that the graph of a function is the set of ordered pairs consisting of an input and the corresponding output.	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f*(*x*) denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation $y = f(x)$.	Use function notation, and evaluate functions for inputs in their domains. Interpret statements that use function notation in terms of a real-world context. *For example, evaluate a given function for an input in its domain and describe the meaning of that input.*	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*
Properties	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal	Interpret the parameters in a linear or simple exponential function in terms of a context. *For example, given points on a graph, construct a model*	Given the graph of a function that models a relationship between two quantities, identify the type of function represented by interpreting key features of graphs. *Functions include: linear, quadratic, basic cubic, absolute value, exponential,*	

	descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*		*and interpret slope and intercepts.*	*logarithmic, reciprocal, square root, and cube root. Key features include: slope (linear), intercepts (linear, quadratic), and general shape of the graph of each function.*	
Proportionality	Solve basic rate and ratio problems. *For example:*	Solve mathematical problems involving simple proportions (single copy of the variable being sought). *For example, solve* $\frac{n}{3}=\frac{1}{2}$ *for n.*	Use proportional relationships to solve multistep ratio and percent problems. *For example: similar triangles, unit conversion, simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error, unit and dimensional analysis.*	Solve proportion problems involving algebraic expressions. *For example, solve* $\frac{x+5}{2}=\frac{4}{5}$ *for x.*	Use proportional relationships to solve multistep ratio and percent problems. *For example: similar triangles, unit conversion, simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error, unit and dimensional analysis.*
Graphing					
Number Line and Coordinate Plane	Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., $-(-3) = 3$, and that 0 is its own opposite.	Find and position integers and other rational numbers on a vertical or horizontal number line diagram.	Graph simple inequalities in one variable on the number line. *For example, solve* $x-5>3$ *for x and graph the solution set.* Represent > and < symbols as open circles, and ≥ and ≤ symbols as closed circles.	Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the	Represent real-world and mathematical problems by graphing points on the coordinate plane (using an appropriate scale), and interpret coordinate values of points in the context of the situation.

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				two axes and the coordinates correspond (e.g., *x*-axis and *x*-coordinate, *y*-axis and *y*-coordinate).	
Modeling Functions	Graph functions expressed symbolically and show key features of the graph. • Graph linear functions and show intercepts. • Graph linear inequalities.	Analyze and solve 2×2 systems of linear inequalities. • Understand that solutions to a system of two linear inequalities in two variables correspond to the regions of intersection of their graphs, because regions of intersection satisfy both inequalities simultaneously.	Graph functions expressed symbolically and show key features of the graph. • Graph quadratic functions and show intercepts, maxima, and minima. • Graph square root, cube root, and piecewise-defined functions, including absolute value functions. • Graph polynomial functions (third degree or less), identifying zeros when suitable factorizations are available, and showing end behavior.	Graph exponential and logarithmic functions, showing intercepts, end behavior, and asymptotes.	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs.
Linear Relation-ships	Given a linear equation, determine whether a given ordered pair is a solution of the equation. Understand that the graph of an equation in two variables is the set of its solutions plotted	Given a linear inequality, determine whether a given ordered pair is part of the solution set.	Identify that a line described by an equation $y = mx$ passes through the origin and that a line described by an equation $y = mx + b$ intercepts the vertical axis at b.	Interpret the slope (rate of change) and the intercept (constant term) of a given linear model in the context of the data.	Translate between representations of linear functions (graph, equation, table of ordered pairs, verbal description) identifying key features (slope, intercepts).

	in the coordinate plane.						
Numbers and Operations							
In Base Ten	Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.	Use place value understanding to round numbers to any place.	Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., 347.392 = 3 × 100 + 4 × 10 + 7 × 1 + 3 × (1/10) + 9 × (1/100) + 2 × (1/1000).	Read, write, and compare decimals to thousandths. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.	Identify patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10. Convert between scientific notation and decimal notation.	Perform operations with numbers expressed in scientific notation, using only positive exponents, including problems where both decimal and scientific notation are used. Interpret scientific notation that has been generated by technology.	Perform operations with numbers expressed in scientific notation, using both positive and negative exponents, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading).
Fractions	Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. • Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.			Compare two fractions, using >, =, and < symbols to record the results of comparisons.	Use decimal notation for fractions with denominators 10, 100, or 1,000. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters;*	Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an	

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	• Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3). Explain why the fractions are equivalent. • Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.*			*locate 0.62 on a number line diagram.*	equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)*	
Rational Numbers and Exponents	Solve simple problems involving whole numbers, fractions, decimals, and percents.	Determine whether a given whole number is prime or composite. Express a composite number as a product of its prime factors. Determine the greatest common factor (GCF) and the least common multiple (LCM) of whole numbers.	Apply and extend previous understandings of multiplication and division and of fractions to multiply, divide, and exponentiate rational numbers. • Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (–1)(–1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. • Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If *p* and *q* are integers, then $-(p/q) = (-p)/q = p/(-q)$. • Apply properties of operations as strategies to multiply, divide, and exponentiate rational numbers. • Convert a rational number to a decimal and a decimal to a rational number.	Solve mathematical problems involving the four operations with non-negative rational numbers. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity).	Solve problems involving finding the whole, given a part and the percent and problems involving finding the part, given a percent and the whole. Estimate to solve real-world problems (e.g., calculating sales tax, calculating interest rates, determining time when distance and rate are known). Assess the reasonableness of answers using mental computation and	Solve real-world and mathematical problems involving the four operations with rational numbers. Solve mathematical problems involving the four operations with rational numbers. Describe situations in which opposite quantities combine to make 0, such as $\left(-\frac{2}{3}x\right)+\left(\frac{2}{3}x\right)$.